

# CHARACTERIZATION OF THE RESONANT AND COUPLING PARAMETERS OF DIELECTRIC RESONATORS FOR NRD-GUIDE FILTERING DEVICES

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## Abstract

Basic material to accurately design millimeter-wave filtering devices using variously-shaped dielectric resonators in Non-Radiative Dielectric waveguide is presented. By developing different approximate and rigorous theoretical methods, the resonant and coupling properties (frequencies, field configurations, unloaded and loaded quality factors, scattering parameters, etc.) are modeled and implemented in an adaptive code. Experimental comparisons through suitable measurements are performed to validate the theoretical and numerical results.

## Introduction

The Non-Radiative Dielectric (NRD) waveguide [1], which consists of a rectangular dielectric rod embedded between metal plates at a distance less than half a free-space wavelength (Fig. 1a), is recently increasing its popularity for applications in communication systems [2]. Millimeter-wave integrated circuitry based on this structure shows typical advantages: due to the specific geometry and operation mode, in NRD devices a strong reduction of losses and interference by radiation from discontinuities, bends and junctions is achieved. In the design of frequency-selective devices, such topology presents favourable integrability with dielectric resonators (DRs) [3], making it possible to implement compact, low-cost and high-performance components.

The theoretical knowledge to precisely carry out these filtering subsystems is anyway quite complex and not extensively explored in the literature [1,2].

The fundamental material, regarding the resonant and coupling properties of NRD DRs with different shapes, is here investigated from both theoretical and experimental viewpoints.

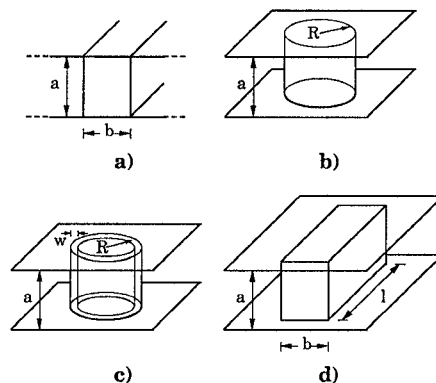


Fig.1 - Non-Radiative Dielectric waveguide and typical shapes of NRD dielectric resonators: a) NRD guide; b) Circular pillbox or disc; c) Ring; d) Parallelepiped or rectangular DR.

## Resonant frequencies, field configurations, and unloaded quality factors

The evaluation of the resonant frequencies of DRs is the first step in filter design. In NRD-integrated technology, the resonators are usually sandwiched between the metal plates and excited by the operating NRD mode ( $LSM_{01}$ ) [1]. Typical shapes of NRD DRs that have been considered are the circular pillbox or disc, the ring, and the parallelepiped (Fig. 1b, c, d).

As is well known, the resonant-mode analysis for circular cylindrical shapes (disc, ring) is achievable through an analytical approach based on field matching, which leads to transcendental eigenvalue equations [4]. For the analysis of the rectangular

shape, numerical methods have to be employed [5]. The relevant mode-field configurations can be also obtained in closed or numerical forms, according to the DR's geometry.

To determine the resonant parameters by varying the geometrical and electromagnetic characteristics of the DRs, we have implemented procedures in a code that can make use of different numerical techniques (e.g., mode-matching, finite elements, transverse resonance), related to the specific geometry of the structure, the desired degree of precision, and so on.

For three different shapes of DRs, that is pillbox, ring and rectangular, we present in Table I examples of resonant-frequency values. Comparisons with suitable measurements (see further for details on the experimental testing) show the excellent fitting of our theoretical results. Regarding the circular geometry, we have carried out a specific investigation on the resonant properties of the Whispering Gallery Modes in NRD DRs, whose utilization presents typical advantages for filtering at millimeter wavelengths [6].

Pillbox WGE modes (p=1, m=1)	Resonant Frequencies (GHz)
n=5	60.463
n=6	63.092
n=7	65.867
n=8	68.770
n=9	71.786
n=10	74.902

Ring-DR modes Theoretical Values (GHz)	Ring-DR modes Experimental Values (GHz)	Error Percentage
10.097	10.105	- 0.08
10.225	10.235	- 0.10
10.257	10.257	0.00
10.568	10.575	- 0.07
10.867	10.867	0.00
11.046	11.052	- 0.06
11.576	11.575	+ 0.01
11.622	11.627	- 0.05

DR Length (mm)	Measured Values (GHz)	TRT Values (GHz)	Error %	EDCM Values (GHz)	Error %	PMT Values (GHz)	Error %	FEM Values (GHz)	Error %
5	11.92	12.53	+ 5.1	11.52	- 3.4	11.87	- 0.4	11.91	- 0.1
7.5	11.54	11.68	+ 1.2	11.18	- 3.1	11.49	- 0.4	11.53	- 0.1
10	11.21	11.21	0.0	10.93	- 2.5	11.18	- 0.3	11.22	- 0.1
15	10.73	10.72	- 0.1	10.61	- 1.1	10.73	0.0	10.80	+ 0.7
20	10.50	10.47	- 0.3	10.42	- 0.8	10.50	0.0	10.54	+ 0.4
25	10.36	10.32	- 0.4	10.30	- 0.6				
30	10.23	10.23	0.0	10.21	- 0.2				
35	10.17	10.17	0.0	10.16	- 0.1				
40	10.13	10.12	- 0.1	10.11	- 0.2				

c)

**Table I** - Theoretical resonant frequencies of different NRD DRs and comparisons with measurements.

a) Pillbox (alumina): spectrum of NRD quasi-TE Whispering-Gallery Modes ( $WGE_{npm}$ ) for angular variation  $n$  between 5 and 10.

Parameters:  $\epsilon_r=9.6$ ;  $a=1$  mm,  $R=4$  mm.

b) Ring (rexolite): theoretical values of resonant frequencies compared with measurements in the 10-12 GHz band.

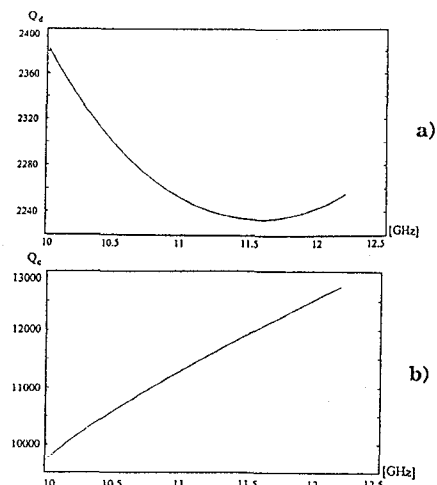
Parameters:  $\epsilon_r=2.53$ ;  $a=12.2$  mm,  $w=10$  mm,  $R=25$  mm.

c) Parallelepiped (rexolite): theoretical and experimental resonant frequencies of the first LSM mode in parallel stub configuration [5], as a function of the stub length  $l$ . TRT: Transverse Resonance Technique, EDCM: Effective Dielectric Constant Method, PMT: Point Matching Technique, FEM: Finite Element Method.

Parameters:  $\epsilon_r=2.53$ ;  $a=12.3$  mm,  $b=10.15$  mm.

Through such numerical methods, we can also derive the amplitudes of the mode components for the different DR shapes.

This modal analysis provides the first information, necessary for the theoretical derivation of the unloaded quality factor  $Q_0$ . In fact, the total stored resonant energy and the losses in the dielectric and in the metal can be calculated through integral expressions in terms of field components. The values of  $Q_0$  clearly depend on the shape of the DRs, the type of the excited mode, and the electromagnetic properties of the materials. Then, by using the previous knowledge on the resonant modes, numerical procedures have been implemented to obtain a straight computation of the unloaded quality factors. Specific behaviors for  $Q_0$  are presented in Fig. 2.



**Fig. 2** - Unloaded quality factors  $Q_0$  of a rectangular NRD DR in silver-plated rexolite, related to dielectric losses  $Q_d$  (a) and to metallic losses  $Q_c$  (b), as a function of frequency. Parameters as in Table I c.

## External and loaded quality factors

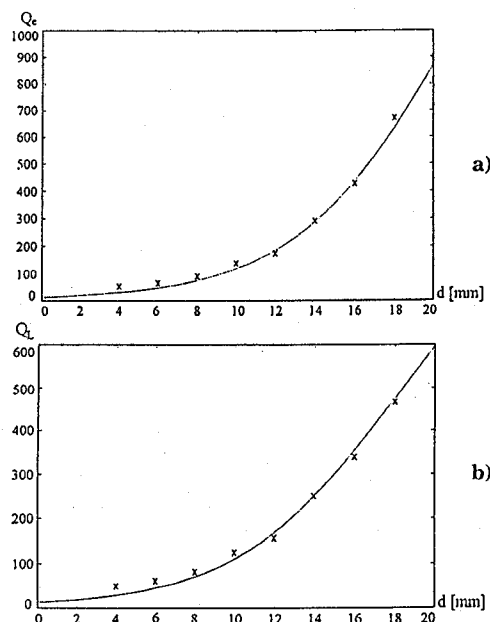
The theoretical evaluation of the external and loaded quality factors, linked to the actual bandwidth of a filter, is a very involved matter, depending on various electromagnetic and geometrical parameters. Information is usually obtained by performing limited experimental investigations or heavy numerical approaches.

A procedure based on a lumped-circuit approach [3] has been here applied, considering NRD DRs of different shapes and acting on various modes. The

interaction has been taken into account by evaluating the electromagnetic-flux link between guide and DRs in terms of mutual inductance or capacitance. Even if approximate, the numerical results of this method appear to be quite accurate, with reference to an experimental investigation. Nevertheless, a rigorous analysis of the coupling effects has been developed too, as explained in the next section.

Measurements have been performed through a network analyzer; for ease of manufacturing and testing, prototypes in rexolite acting in X band have been realized, although the best utilization can be achieved at millimeter wavelengths. Concerning the quality factors, comparisons using a typical transmission method [3] show a very good agreement between theoretical and experimental results. Behaviors of the external and loaded quality factors versus the coupling distance are shown in Fig. 3.

Values of the bandwidths and of the coupling coefficient, given by the ratio between the unloaded and the external quality factors, can be also easily determined.



**Fig. 3** - Theoretical and experimental data of the external quality factor  $Q_e$  (a) and of the loaded quality factor  $Q_L$  (b) for a rectangular DR in bandstop-filter parallel configuration, as a function of the coupling distance  $d$ .

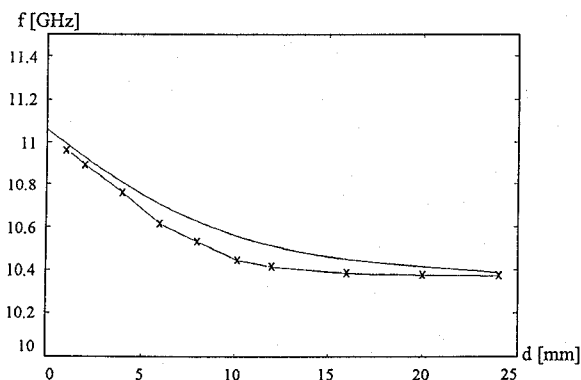
Parameters:  $\epsilon_r=2.53$ ;  $a=12.1$  mm,  $b=10.05$  mm,  $l=30$  mm.

Moreover, the coupling distance strongly influences the band center-frequency shift, as is evident in the wide-band measurements of the

transmission scattering parameter. Here a simple method, based on a resonant equivalent-circuit approach, has been worked out in order to predict this behavior. In Fig. 4 we show typical theoretical and experimental results.

Similar methods have been also used to predict the coupling effect between two or more DRs.

The optimum coupling distance as regards the filter insertion loss has been also investigated by extending an approximate procedure [7], which is particularly suitable for the design of NRD-ring directional filters.



**Fig. 4** - Theoretical and experimental behavior of the resonant-peak frequency as a function of the coupling distance  $d$  for a rectangular DR. Parameters as in Fig. 3.

### A surface-integral-equation approach for the coupling between NRD-guide and DRs

A rigorous approach to evaluate the coupling parameters between dielectric waveguides and resonators has been also carried out on the basis of the Boundary Element Method (BEM).

The interaction between the structures has been formulated as a problem of scattering from obstacles in waveguides [8]. In order to get the total electromagnetic field, given by the sum of an incident and a scattered part, a couple of two-dimensional non-homogeneous integral equations is derivable. The unknowns are the equivalent currents linked to the tangential components of the magnetic and electric fields on the surface of the resonator; the integral kernels are given by the electric and magnetic dyadic Green's functions for the regions outside and inside the boundary of the resonator [9].

Therefore, we have calculated the dyadic Green's functions for the NRD guide, in terms of a series of

guided modes and an integral of radiation modes. Then, the expressions for the Green's functions of the complementary structure can be properly chosen according to the DR's shape. The integral-equation solution has to be obtained numerically, for instance through the method of moments or point matching techniques. By considering a cylindrical DR, Galerkin's method has been applied by expanding the unknown fields as a sum of suitable basis functions, whose coefficients have been numerically computed.

An analytico-numerical formulation for the scattering parameters of a NRD-DR bandstop configuration has been then reached and implemented. Here we present the expression obtained for the transmission parameter, as a function of electromagnetic and geometrical quantities (frequency, permittivity, dimensions, coupling distance, etc.):

$$S_{21} = 1 - \frac{aR\pi}{2} A_{01} \cos(k_{xe}b/2) e^{-j k_{x0}(d-b/2)} \sum_{n=0}^N \left\{ \frac{\pi |k_{x0}|}{ja\omega\epsilon_0} (A_n^e P_n + A_n^o Q_n) + \right. \\ \left. + B_n^e \left[ \frac{(\pi/a)^2 + \beta^2}{2j\omega\epsilon_0} (P_{n+1} + P_{n-1}) - j \frac{\beta |k_{x0}|}{2j\omega\epsilon_0} (Q_{n+1} - Q_{n-1}) \right] + \right. \\ \left. + B_n^o \left[ \frac{(\pi/a)^2 + \beta^2}{2j\omega\epsilon_0} (Q_{n+1} + Q_{n-1}) + j \frac{\beta |k_{x0}|}{2j\omega\epsilon_0} (P_{n+1} - P_{n-1}) \right] + \right. \\ \left. - j\beta (C_n^e P_n + C_n^o Q_n) + \left[ -\frac{D_n^e}{2} (Q_{n+1} + Q_{n-1}) + \frac{D_n^o}{2} (P_{n+1} + P_{n-1}) \right] \right\}$$

where  $A_n, B_n, C_n, D_n$  are the expansion coefficients obtained by Galerkin's method ( $n$  is linked to the azimuthal periodicity); the apices  $e/o$  represent even/odd harmonic solutions;  $A_{01}$  is the LSM<sub>01</sub> incident-field amplitude;  $\beta$  the propagation constant;  $k_{xe}, k_{x0}$  the wavenumbers in the dielectric and air, respectively, along the transverse direction parallel to the metal plates;  $R$  is the pillbox radius;  $a, b$  the height and the width of the NRD guide, respectively;  $d$  the NRD-DR coupling distance; moreover, we have defined:

$$P_n = I_n(\gamma R_1) \begin{cases} (-1)^{n/2} \cosh n\phi \\ j(-1)^{(n-1)/2} \sinh n\phi \end{cases} \begin{cases} \text{even } n \\ \text{odd } n \end{cases} \\ Q_n = I_n(\gamma R_1) \begin{cases} -j(-1)^{n/2} \sinh n\phi \\ (-1)^{(n-1)/2} \cosh n\phi \end{cases} \begin{cases} \text{even } n \\ \text{odd } n \end{cases} \\ \gamma = [(\pi/a)^2 - k_0^2]^{1/2}, \quad \phi = \frac{1}{2} \ln \frac{|k_{xd}| + \beta}{|k_{xd}| - \beta}$$

where  $I_n$  is the  $n$ -order modified Bessel function of first kind.

Even if a large amount of pre-processing is needed and numerical computations are heavy, such an integral approach allows us to accurately solve the coupling problem of these structures.

## Conclusion

An extensive analysis of the resonant and coupling characteristics of variously-shaped dielectric resonators in NRD waveguide has been carried out, by means of numerical implementations of different theoretical approaches. A number of comparisons through measurements on basic filtering devices allow us to confirm the validity of the presented models.

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